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Absolute Median Deviation Based a Robust Image Segmentation Model

Tahir Ullah *
Department of Mathemetics, University of Peshawar.

Lutful Mabood *
Department of Mathemetics, University of Peshawar.

Haider Ali *
Department of Mathemetics, University of Peshawar.

Noor Badshah *
Department of Basic sciences, UET Peshawar.

ABSTRACT

The level set methods for image segmentation are usually embedded in variation framework. In the variation framework for image segmentation, a nominee level set function for capturing valuable edges and boundaries is modeled as a minimum of a well-designed functional. The design of the functional mainly bases on the image data embedded. The image data may either be the image gradients or image statistical information. The insufficient and incorrect amount of image data embedded in the functional may leads to inaccurate level set function and consequently incorrect image segmentation. In the recent work by X. F. Wang et al designed a functional (LCV) consisting mainly two terms. One of the terms was responsible for providing image global information and the other term for image local information. In continuous sense the L2 norm and in discrete sense, the statistic variance was utilized for this purpose. Although LCV model works well in some tests but it is not that much efficient and robust with respect to CPU timing and quality of detection in noisy images. To segment noisy images robustly, we propose a new model based on the concept of L1 and in discrete sense, absolute median deviation. The Experimental tests validate that the use of L1 or the statistics absolute median deviation gives best results in noisy images both in terms of quality and timing in contrast with LCV model.

Keywords : Level Set Methods, Absolute Median Deviation, Functional Minimization, Partial Differential Equations.

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[1] Tahir ullah : tahirkhan264@gmail.com
[2] Haider Ali : haider_uop99@yahoo.com
[3] Noor Badshah : noor2knoor@gmail.com
[4] Lutful Mabood : lathizm@gmail.com

1. INTRODUCTION

Image segmentation is an important branch of computer vision. Its aim is to extract meaningful lying objects in images, either by dividing images into contiguous semantic regions, or by extracting one or several objects more specific in images, such as medical structures. In general, image segmentation task is very difficult to achieve it since natural images are diverse, complex and the way we perceive them, vary according to individuals (J. Shah, 1989). Various models have been developed for image segmentation tasks. Mumford- Shah functional minimization (J. Shah, 1989), region growing and emerging (L. Bischof ,1994), watershed algorithms (Soiled, 1994),and minimum description length criteria are the examples(Y. Leclerc,1990). For edge detection most of the models use an edge detector function which mainly depends on the gradient of a given image (C. Gout. Geodesic, 2008). Region based segmentation models are those models in which objects are detected by detecting their occupied optimal regions. Usually in the models, some special terms are added which work to detect regions in an image. These terms are named as region detectors or ideality term. The literature is rich in such models in which the statistic, variance is usually used as a ideality term. This idea of using variance as an ideality term initiated due to the work by David Mumford and Jayant Shah. Most influential and the best known among the segmentation models is the Mom ford-Shah (MS) image segmentation model (J. Shah, 1989). Minimization of the MS functional leads to the segmentation of an image which is much closer to the original image while containing short object boundaries and low variation. In this formulation, for a given image zwe search for an image u: ? ? R and a set of edges K ??, minimizing

$$F(u,K)^{MS} = \int_{\Omega}(u-z)^2\,dxdy + \alpha\int_{\Omega-K}|\nabla u|^2\,dxdy + \beta\int_K ds.$$

The first term in above model is called ideality term which keeps the solution image u as much closer as possible to the given image z, keeping remaining constraints (terms) in consideration. The second term is called regularize which helps to give the solution image u smooth in the region ? - K inside the prominent edges. The third is the length term, which helps to shorten edges as possible so that edges look straight, where a and ß are positive (tuning) parameters. With this mechanism, whenever an image u0 is segmented, an image u is obtained with smooth regions and clear sharp edges. In other words, a cartoon version of a given image is obtained. The snake model of aims to find the segmentation curve C (in parametric form with C(s): [0, 1] ? R2) by solving the following minimization problem (D. Terzopoulos, 1988).

$$F(u,k)^{MS} = \int_0^1 \alpha\,|C'(s)|^2 + \beta\,|C''(s)| - \lambda\,|\nabla z(C(s))|^2\,ds.$$

The geodesic active contour model [3] proposed to find C by solving

$$\min_{C} F(C) = \int_0^1 | C'(s) |^2 \; g(| \nabla z(C(s)) |) ds,$$

Where g is an edge detection function e.g. for some p = 1 and a Gaussian Gs (x, y)

$$g(| \nabla z(x,y) |) = \frac{1}{1 + | \nabla G_\sigma(x,y) * z(x,y) |^p}$$

Because all these classical snakes and active contour models mainly depend on the edge function g which uses the image gradient |?z|, to stop the curve evolution, these models can detect only objects with edges defined by gradient. In practice, the discrete gradients are bounded and then the stoppings function in never zero on the edges. If the image is very noisy, then the isotropic smoothing Gaussian has to be strong, which will smooth the edges as well. In contrast, the Chan-Vese (CV) active contour model without edges proposed indoes not use the stopping edge function g to find the edge of an object. Instead the stopping term is based on Mumford-Shah segmentation techniques(L. A. Vese,2001).

2. THE CHAN-VESE MODEL (CV)

Chan and Vese [4] proposed an active contour model which can be considered as a special case of the Mumford-Shah model [12]. For a given image z(x, y) in domain ?, the CV model is formulated by minimizing the following energy functional:

$$F^{CV} = \lambda_1 \int_{inside(\Gamma)} | z - c_1 |^2 \, dx + \lambda_2 \int_{outside(\Gamma)} | z - c_2 |^2 \, dx,$$

Where c1 and C2 are two constants which are the average intensities inside and outside the contour G respectively, with the level set method, we assume

$$\begin{cases} \Gamma = \{(x,y) \in \Omega : \phi(x,y) = 0\}, \\ inside(\Gamma) = \{(x,y) \in \Omega : \phi(x,y) > 0\}, \\ outside(\Gamma) = \{(x,y) \in \Omega : \phi(x,y) < 0\}. \end{cases}$$

By incorporating the length energy term into Eq. (4) and minimization leads to the following evolution equation

$$c_1(\phi) = \frac{\int_\Omega z H_\varepsilon(\phi) dxdy}{\int_\Omega H_\varepsilon(\phi) dxdy}, \; c_2(\phi) = \frac{\int_\Omega z(1 - H_\varepsilon(\phi)) dxdy}{\int_\Omega (1 - H_\varepsilon(\phi)) dxdy},$$

Where µ = 0, ?1 >0, ?2 >0 are fixed parameters, µ controls the smoothness of zero

$$\frac{\partial \phi}{\partial t} = \delta(\phi) \left[\mu \nabla . \left(\frac{\nabla \phi}{| \nabla \phi |} \right) - \lambda_1 (z - c_1)^2 + (z - c_2)^2 \right],$$

Level set and ?1, ?2 control the image data driven force inside and outside the contour respectively. H (?) is the Heaviside function and d (?) is the dirac delta function. Usually, regularized Heaviside and delta is used as given by. Since CV model is based on the assumption that image intensities are statistically homogeneous in each region, and Therefore it fails to segment images having inhomogeneous intensity regions. Although, the CV model has some advantages such as edge detection without gradient, robustness in topological.

3. LOCAL CHAN-VESE MODEL (LCV)

Changes and simple numerical implementation, however it has many weaknesses as well. For example the CV model usually detects incomplete regions/objects of variable intensities. On the other hand, the CV model in discrete sense uses arithmetic mean which is largely affected by the presence of outliers. Thus theoretically the performance of the CV model cannot be trusted in noisy images. To modify the CV model for images with intensity in homogeneity so that an object with inhomogeneous intensities can be completely detected, X-F. Wang et al, proposed a model by incorporating their local statistical functional in the CV energy functional. The proposed regularized energy functional of the LCV model in level set formulation is given by (Wang et al, 2010):

$$F_\varepsilon(c_1, c_2, d_1, d_2) = \mu\int_\Omega \delta_\varepsilon(\phi)|\nabla\phi|\,dxdy + \int_\Omega \frac{1}{2}(|\nabla\phi|-1)^2\,dxdy$$

$$+\lambda_1\int_\Omega (z-c_1)^2 H_\varepsilon(\phi)dxdy + \lambda_1\int_\Omega (z-c_2)^2(1-H(\phi))dxdy,$$
$$+\lambda_2\int_\Omega (z^*-d_1)^2 H_\varepsilon(\phi)dxdy + \lambda_2\int_\Omega (z^*-d_2)^2(1-H(\phi))dxdy,$$

Where µ, ?1, ?2 are constants and are used for assigning different weights and (x, y) denotes the difference image gk *z(x, y) - z(x, y), where gk is an averaging convolution operator of window size k × k.

The first term $\int_\Omega \delta_\varepsilon(\phi)|\nabla\phi|\,dxdy$ of the energy functional given in (8) is a regularized term that helps to maintain the smoothness of the active curve.

$$H_\varepsilon(\varphi) = \frac{1}{2}\left(1+\frac{2}{\pi}\arctan\left(\frac{\varphi}{\varepsilon}\right)\right),\quad \delta_\varepsilon(\varphi) = \left(\frac{\in}{\pi(\in^2+\varphi^2)}\right).$$

The second term is a global term which helps the active contour to capture the main structure of objects/regions an image.

$$\lambda_1\int_\Omega (z-c_1)^2 H_\varepsilon(\phi)dxdy + \lambda_1\int_\Omega (z-c_2)^2(1-H(\phi))dxdy,$$

$$\lambda_2\int_\Omega (z^*-d_1)^2 H_\varepsilon(\phi)dxdy + \lambda_2\int_\Omega (z^*-d_2)^2(1-H(\phi))dxdy.$$

The third term is a local term which contributes in detecting small and valuable details. In similar lines to section 2, minimization of F? (?, c1, c2, d1, d2) leads to the following solutions,

$$c_1(\phi)=\frac{\int_\Omega z(x,y)H_\varepsilon(\phi)dxdy}{\int_\Omega H_\varepsilon(\phi)dxdy},\ c_2(\phi)=\frac{\int_\Omega z(x,y)(1-H_\varepsilon(\phi))dxdy}{\int_\Omega (1-H_\varepsilon(\phi))dxdy},$$

And,

$$d_2(\phi)=\frac{\int_\Omega z^*(x,y)(1-H_\varepsilon(\phi))dxdy}{\int_\Omega (1-H_\varepsilon(\phi))dxdy},\ d_2(\phi)=\frac{\int_\Omega z^*(x,y)(1-H_\varepsilon(\phi))dxdy}{\int_\Omega (1-H_\varepsilon(\phi))dxdy},$$

4 .THE PROPOSED MODEL (AMD)

In contrast with CV model, the LCV model works well in many examples effected with image in homogeneity problems (Wang et al, 2010). The local image data functional in the LCV model helps in capturing minute details and detecting objects/regions with intensity in homogeneity better, in contrast with the CV model. However in noisy images, the performance of the LCV model is worse than the CV model. The LCV model uses arithmetic mean as well as local data functional. The local data functional of LCV model detects noise by considering it valuable minute details which is of course a big drawback of the LCV model. Based on the concept of absolute median deviation (AMD), we introduce a new type of ideality term given by,

$$\lambda_1\int_{outside(\Gamma)}|z-med_{G1}|\,dxdy+\lambda_2\int_{inside(\Gamma)}|z-med_{G2}|\,dxdy.$$

Where and are two constants which are the median intensities inside and out- side the contour G respectively. The ideality term in (10), in continuous sense, can be interpreted as L1 norm based ideality term. This ideality term is popular in the literature for its efficient performance in noisy images, accurate edge detection and benefits such as global minimization (Q. X. Wu, 1995). Here for a discrete image z, the new model may be explained as follows. Denoting the image intensity at position (i, j) as zi,j , the absolute median deviation is defined by

$$MD(z)=\frac{1}{N}\sum_{i,j}|z_{i,j}-Median(z)|$$

Where Median (z) denotes the median intensity of a given image the average median is very less sensitive in the presence of outliers (noise), in contrast with mean. Based on the above discussion, we design the following global image data ideality term (11) which will be responsible for fast detection and will work in alliance with local image data ideality term (12) to detect valuable minute details without capturing noise.

$$F_1 = \int_{outside(\Gamma)} |z - med_{G1}| \, dxdy + \lambda_2 \int_{inside(\Gamma)} |z - med_{G2}| \, dxdy.$$

And,

$$F_2 = \int_{outside(\Gamma)} |(g_k * z - z) - med_{L1}| \, dxdy + \int_{inside(\Gamma)} |(g_k * z - z) - med_{L2}| \, dxdy.$$

Where gk an averaging convolution operator of window size k × k, medG1, medG2, medL1 and medL2 are the median intensities of the image z and difference image (gk *z - z)

5.1 PROPOSED MODEL (AMD)

Inside and outside G, respectively, Thus we propose the following energy functional by denoting the difference image (gk *z - z) with ,

$$F = \mu length(\Gamma) + \lambda_1 F_1 + \lambda_2 F_2.$$

Thus in level set formulation we have,

$$F_\in(\phi, m_1, m_2, d_1, d_2) = \mu \int_\Omega \delta(\phi) |\nabla \phi| \, dxdy + \int_\Omega \left[\lambda_1 |z - med_{G1}| + \lambda_2 |z^* - med_{L1}| \right] H(\phi) dxdy$$

$$+ \int_\Omega \left[\lambda_1 |z - med_{G2}| + \lambda_2 |z^* - med_{L2}| \right] (1 - H(\phi)) dxdy$$

Where µ, ?1 and ?2 are constants and are used for assigning different weights. Consider the following regularized minimization problem

$$\min_{(\phi, med_{G1}, med_{G2}, med_{L1}, med_{L2})} F_\in(\phi, med_{G1}, med_{G2}, med_{L1}, med_{L2}),$$

Where

$$F_\in(\phi, med_{G1}, med_{G2}, med_{L1}, med_{L2}) = \mu \int_\Omega \delta(\phi) |\nabla \phi| \, dxdy + \int_\Omega \left[\lambda_1 |z - med_{G1}| + \lambda_2 |z^* - med_{L1}| \right] H(\phi) dxdy$$

$$+ \int_\Omega \left[\lambda_1 |z - med_{G2}| + \lambda_2 |z^* - med_{L2}| \right] (1 - H(\phi)) dxdy$$

Where H? is regularized Heaviside function given in (L. A. Vese, 2001). Thus to minimize the above functional, at each iteration median values are computed and updated. Minimizing F? with respect to ? yields the following Euler-Lagrange equation for :

$$\begin{cases} \delta_\varepsilon(\phi) \left[\mu div \left(\frac{\nabla \phi}{|\nabla \phi|} \right) \right. \\ \left. -\lambda_1 |z - med_{G1}| - \lambda_2 |z^* - med_{L1}| + \lambda_1 |z - med_{G2}| + \lambda_2 |z^* - med_{L2}| \right] = 0 \quad \text{in } \Omega, \\ \frac{\partial \phi}{\partial \vec{n}} = 0 \ in \ \Omega \end{cases}$$

Where exterior unit normal to the boundary ??, and the normal derivative of ? at the boundary, The above PDE may be considered as a steady state form of the following evolution equation.

$$\frac{\partial \phi}{\partial t} = \delta_{\varepsilon}(\phi)\left[\mu div\left(\frac{\nabla \phi}{|\nabla \phi|}\right) \right.$$

$$\left. -\lambda_1 |z - med_{G1}| - \lambda_2 |z^* - med_{L1}| + \lambda_1 |z - med_{G2}| + \lambda_2 |z^* - med_{L2}| \right] \quad \text{in } \Omega$$

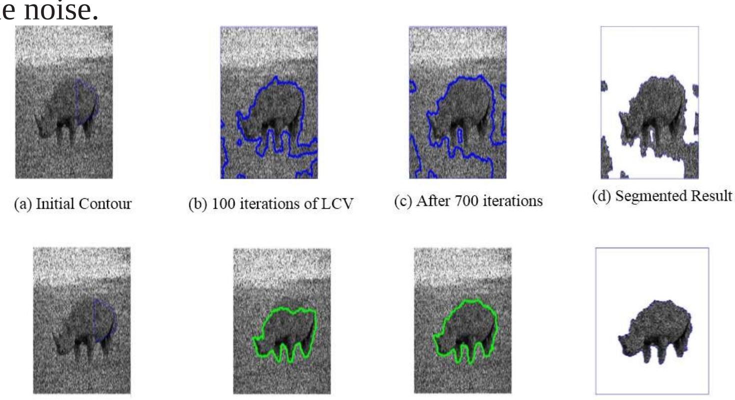
$$\phi(x, y, t) = \phi_0(x, y, 0), \quad in \ \Omega.$$

5. COMPARISON RESULTS

In this section we present some experimental results. The first row of each experiment indicates the result of LCV model, while the second row shows the result of AMD model. In the beginning, the LCV and AMD model are tested on noisy real images. In figure 1 the performance of the proposed AMD model can be observed in terms of accurate detection and in terms of number of iterations. The proposed AMD model detected objects/regions in the given image 1, accurately in 500 iteration. In addition, the segmented result of figure 2 ensures the quality of detection of the proposed method. The same interpretations and conclusions can be obtained from the rest of others figures by observing 3 and 4. In figure 3, a noisy medical image can be seen. Figures 3 and 4 are displaying the efficiency of the LCV model and AMD model respectively. It can be clearly interpreted from these results that AMD is efficient in accuracy and robustness

CONCLUSION

In this paper a new model is proposed which overcome the deficiency of LCV model, and capture important minute details other than noise. It used median and absolute deviation to handle outliers i.e. noise, the proposed model work very well in those images which are effected by speckle noise.



(a) Initial Contour (b) 100 iterations of LCV (c) After 700 iterations (d) Segmented Result

(e) Initial Contour (f) 100 iterations of AMD (g) After 500 iterations (h) Segmented Result

Fig. 1: A test of segmenting real noisy image. The AMD model completed the task in 500 iterations whereas incomplete result of the LCV model in 700 iterations is clearly exhibited. The parameters used for AMD model are; mue = 0.0011 × 255, ?1 = 1 and ?2 = 0.00001.

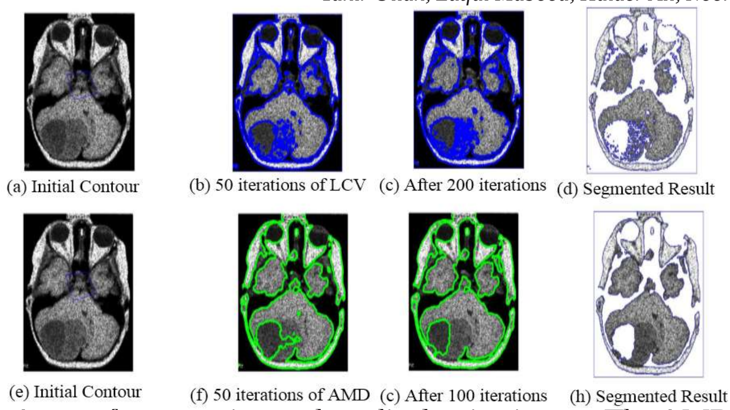


Fig. 2: A test of segmenting real medical noisy image. The AMD model completed the task in 100 iterations whereas incomplete result of the LCV model in 200 iterations is clearly exhibited. The parameters used for AMD model are; mue = 0.001×255, ?1 = 1 and ?2 = 0.1

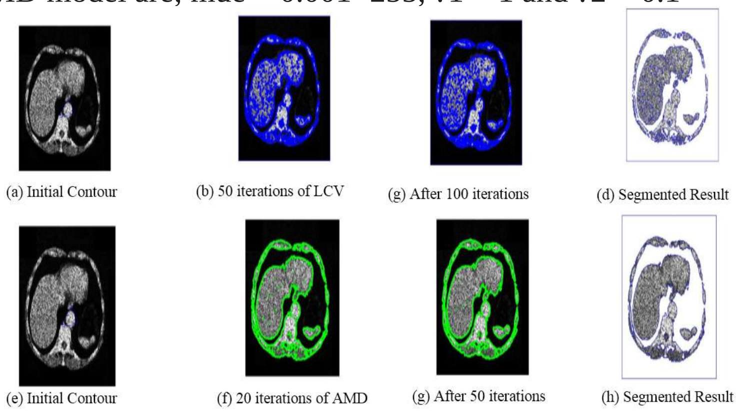


Fig. 3: A test of segmenting real medical noisy image. The AMD model completed the task in 50 iterations whereas incomplete result of the LCV model in 100 iterations is clearly exhibited. The parameters used for AMD model are; mue = 0.0001 × 255, ?1 = 1 and ?2 = 0.000001

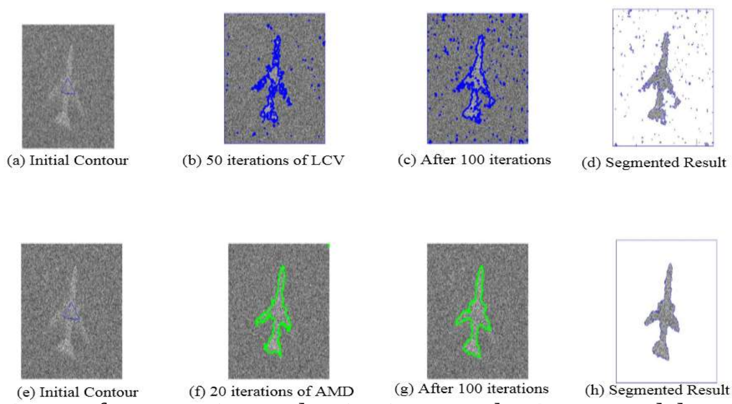


Fig. 4: A test of segmenting real noisy image. The AMD model completed

the task in 100 iterations whereas incomplete result of the LCV model in 100 iterations is clearly exhibited. The parameters used for AMD model are; mue = 0.0005 × 255 , ?1 = 1 and ?2 = 0.00001

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