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COMPARATIVE ANALYSIS OF LMS (LEAST MEAN SQUARE) AND RLS (RECURSIVE LEAST SQUARE) FOR ESTIMATION OF FIGHTER PLANE'S MATHEMATICAL MODEL

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ABSTRACT

Estimating of complex system in real time environment is necessary for designing a precise controller to operate the given system. In this paper we have compared both Least Mean Square (LMS), and Recursive Least Square to identify which one of them gives more approximate estimation. Basically LMS is based on the set of Adaptive filters tends to reduce the error over time between the actual system response and the desired system response, on the other hand we have RLS that is capable of forgetting the previous acquired data and focus on recent main stream data. We have considered a model of a fighter plane because of its complex and unpredicted movements to estimate the model at its follows. Both the algorithms are complete and useful, but their usage may vary according to the application.

Key Words: System Identification, Least Mean Square (LMS), Recursive Least Square (RLS), Modeling Of Fighter Plane.

INSPEC Classification : A9555L, A9630, B5270

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1. INTRODUCTION

Aircraft formation is identified to be prone to nonlinear occurrence, particularly nowadays as they are lighter in weight and more flexible (J.P. No¨el ET.AL, 2005). The major setback in the formation of flight control system is the designing of uncertain parametric variation of any aircraft (Eugene A. Morelli, 2006). We have considered the model of F-16 (Morelli, E.A, 1998), that it is an example of nonlinear model. Our work here presents a viability study of technique for identification and estimation of uncertain parameters of fighter aircraft. The major aim of this research is to investigate the possibility of a method through which coefficient identification and estimation becomes feasible.

Jet consists of various model parameters as well as one or multiple inputs (Morelli, E.A, 1998). In most cases, the target is to determine a structure of model that is not only compact, but still has ample complexity to determine all the nonlinearities (Klein, V. and Morelli, E, 2006). Previously, a method was developed to estimate input and output flight data through frequency response estimation which was not only time consuming but costly too (Tischler, M. and Remple, R., 2011). Another technique has been developed to estimate the aerodynamic parameters without the involvement of airflow angles, while this method offered reduction of cost in testing of flight, insinuations for safety of aircraft were noted (Eugene A. Morelli, 2012). One approach observed for parameter estimation is nonlinear sliding surface control which provided fairly accurate estimation but was unable to guarantee stability or convergence (Gurbacki, Phillip, 2010).

An additional process to estimate the parameters is to consider that the dynamic model include the linear structure with respect to time variant parameters to explain the changes occurring in flight condition; however the major problem afflicted here in parameter estimation is by noise. Perfectly estimating the aerodynamic parameters is vital for control systems that require the estimated parameters as inputs.

Our paper offers calculation of aerodynamic parameters while proving to be stable, convergent as well as robust. The parameter estimator explored here is Least Mean Square (LMS) and Recursive Least square (RLS). LMS having applications from economy growth to aerodynamics is considered to be extremely popular for adaptive system identification (Gurbacki, Phillip, 2010). LMS include an iterative process that formulates consecutive modification to main vector in the direction of the negative of the gradient vector that ultimately escort to the minimum mean square error. This estimator has proven to be superior adaptive estimator.

The main focus of our research is the three dimensions, in which the flight freely rotates, i.e. pitch (lateral), yaw (normal) and roll (longitudinal). The mentioned axes progress with the aircraft while rotating relative to the Earth (Gui and Adachi, 2013). The rotations are created due movement of control surfaces that offers variation in distribution of the overall aerodynamic force with respect to the centre of gravity of the aircraft. These axes are considered to be symmetrical geometrically in spite of mass distribution of jet (Eleftherios Giovanis, 2008).

In this paper section II describes the fighter plane dynamics which includes fighter plane dynamics, coordinate system and equation of motion. Section III is our estimation algorithm; least mean Square. Discussions and results are covered in section IV.

2. LITERATURE REVIEW

2.1 Fighter Plane Dynamics

The dynamics for fighter plane can be estimated by using the various methods including least square estimation, recursive least square estimation and by neural networks. Here we have represented the mathematical model of fighter plane in order to get acquainted with the dynamics of the fighter jet (B. Windrow ET.AL, 1976).

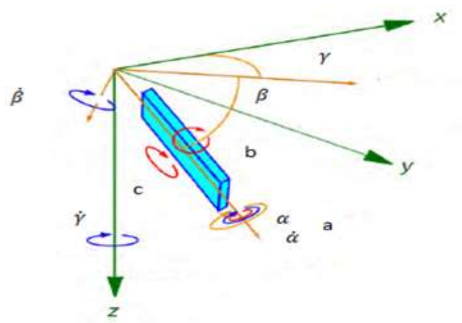

Figure 1: Mathematical model of Fighter Plane

2.2 Coordinated system of Plane

As previously discussed, the model of an aircraft is nonlinear that its parameter varies with respect to time. Here in this section we are intended to derive mathematical equations of it. Before describing equation of motion it is necessary to declare some frame of reference to describe motion in (S. D. Stearns, 1985). The most common reference frames are G_E, commonly called earth fixed reference frame and G_B, called body fixed reference frame. Inside earth fixed reference frame there are Y_E and W_E. The prior points to the center of earth whereas later one points in some arbitrary direction. Essentially, the purpose of earth fixed frame is to describe orientation and position of aircraft. Whereas body fixed frame describes orientation and center of gravity and parameters K_B points forward through nose, L_B axis through the star board right wing, and M_B axis downwards (S. D. Stearns, 1985).

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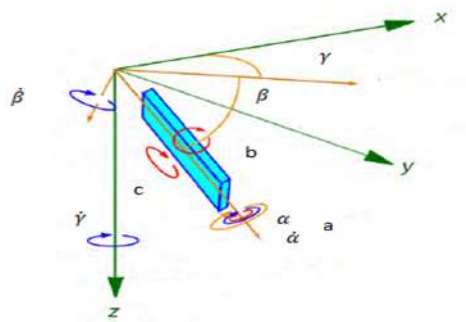

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2.3 Description of variables

Before pursuing to the equations of motions, some preliminary assumptions have been taken into account. A valid assumption for fighter plane is that the air craft is rigid body which reflects that on any two points or within the frame remains fixed with respect to each other. Also, it is tacit for control design of aircraft that earth is flat, none rotating regarded as an inertial reference but not valid for inertial guidance system. The mass is considered constant during which the motion of aircraft is under consideration and fuel consumption is neglected during this time. It is necessary to apply Newton's motions law on this assumption. Similarly, the symmetry in mass distribution of aircraft is relative to K_BOM_B, which means that product of inertia V_{xy} and V_{yz} equals to zero (S. Haykin, 2002).

As a result of mentioned assumptions the aircraft's motion has 6 degree of freedom (DOF) rotation and translation in dimensions; position, orientation, velocity and angular velocity over time describe aircraft dynamics (M. G. Bellanger, 2011).

$B_E=(A_E,B_E,C_E)^T$ position vector expressed in an earth fixed co-ordinate system.

$\delta=(\alpha,\beta,\gamma)^T$ Orientation vector where as α = rollangle, β = pitch angle and γ=yaw angle.

$w = (a,b,c)^T$ Angular velocity where a = roll angular velocity, b=pitch angular velocity and c=yaw angular velocity.

2.4 Equations Of Motions

2.4.1 Euler angle rates

Euler angle rates and body axis rates, orthogonal body axis angular rate vector:

$$w_a=\begin{bmatrix} w_c \\ w_d \\ w_e \end{bmatrix}_a=\begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad (1)$$

Non-orthogonal vector of Euler's angle:

$$\delta=\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \quad (2)$$

Euler angle rate vector:

$$\dot{\delta}=\begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix}\neq\begin{bmatrix} w_c \\ w_d \\ w_e \end{bmatrix}_a \quad (3)$$

Relationship between Euler's angle Rates and body axis Rates:

$$\begin{bmatrix}\dot{\alpha}\\ \dot{\beta}\\ \dot{\gamma}\end{bmatrix}=\begin{bmatrix}1 & \sin\alpha\tan\beta & \cos\alpha\tan\beta\\ 0 & \cos\alpha & -\sin\alpha\\ 0 & \sin\alpha\sec\beta & \cos\alpha\sec\beta\end{bmatrix}\begin{bmatrix}a\\ b\\ c\end{bmatrix} \quad (4)$$

2.4.2 Rigid Body Equations of Motion

Translational position:

$$B_E=\begin{bmatrix}A\\ B\\ C\end{bmatrix}_E \quad (5)$$

Angular position:

$$\delta_E=\begin{bmatrix}\alpha\\ \beta\\ \gamma\end{bmatrix}_E \quad (6)$$

Whereas inertia matrix of the aircraft is given by

$$Q_{ab}=\begin{vmatrix}Q_{xx} & -Q_{xy} & -Q_{xz}\\ -Q_{yy} & Q_{yy} & -Q_{yz}\\ -Q_{xz} & -Q_{yz} & Q_{zz}\end{vmatrix} \quad (7)$$

Where,

$$\dot{\alpha}=a+(b.\sin\alpha+c.\cos\alpha)\tan\beta \quad (8)$$

Aerodynamic and thrust moment,

$$MB=\begin{vmatrix}Gaero & Gthrust\\ Haero & Hthrust\\ Iaero & Ithrust\end{vmatrix} \quad (9)$$

Rate of change of Angular velocity,

$$a' = (QzzG+QxzI-\{Qxz(Qyy-Qxx-Qzz)a+[Qxz+Qzz(Qzz-Qyy)c\}b)/(QxxQzz-Qxz) \quad (10)$$

$$b' = 1/Qyy[H-(Qxx-Qzz)ac-Qxz(p^2-r^2)] \quad (11)$$

$$c' = [Qxz+ QxxN-\{Qxz(Qyy-Qxx-Qzz)r+[Qxz^2+Qxx(Qxx-Qyy)]a\}b)/QxxQzz-Qxz^2] \quad (12)$$

3. ESTIMATION ALGORITHM

Good estimates are extremely important for control schemes. For stochastic systems, parameters having uncertainties and time delays, robust control and filtering has gained significant amount of research work. With fast developments of control systems, beneficial efforts for flexible and effective models have been made. A motivation to develop adaptive control aroused to handle parametric uncertainties greater than those which can be handled by robust control. Over the last few decades, adaptive methods for controlling and identification of dynamical linear time invariant systems have been developed, utilizing unknown parameters.

Various researches and literature is present in this field, describing methods that are robust as well as stable with small uncertainty in plant parameters or the parametric changes occur slowly with respect to time. However, in certain fields like neuroscience, economics, biology and medicines, etc there is a rapid change in parameters with time. Therefore, the solutions presented in are inadequate to cope with the variations. There is a need of new methods for identifying and controlling unknown systems quickly. A realistic control design must be robust, stable, as well as maintains performance with respect to uncertainties in plant such as bounded dynamics. Unknown values of the physical variables and big parametric uncertainties present in plant dynamics should be handled by the controlled design. It is observed that in past few years, large parametric errors may cause oscillatory and transient response of adaptive system and constant efforts are made to improve.

3.1 Least Mean square (LMS)

Least mean squares (LMS) algorithms are a set of adaptive filter used to imitate a preferred filter by finding the filter coefficients that relate to producing the least mean squares of the error signal that is, the difference between the desired and the actual signal.

The LMS is an exploration algorithm in which a generalization of the gradient vector calculation is made promising by suitably adjusts the purposed function [16]. The LMS algorithm is extensively used in a variety of application of adaptive filtering due to its computational ease [17] [18] [19].The LMS algorithm is by far the mainly used algorithm in adaptive filtering for numerous bases like stability when executed with set precision arithmetic, slow convergence and strong performance against unlike signal environment.

The least mean squares (LMS) algorithms fine-tune the filter coefficients to lessen the cost function. As contrast to recursive least squares (RLS) algorithms, the LMS algorithms do not engage with any matrix operations. Therefore, the LMS algorithms require less computational resources and memory than the RLS algorithms and implementation is also less complex than RLS.

For the expectation process, an increasingly accepted method is least square estimation for planting bounded rationality in a model. The least square method entails to the agents included in the model to utilize data generated through the model so that least square foretells prospect variables and that the forecasted variables are revised each time a new data update is available. In every period of the model, all agents utilize same forecasts of least squares' expected values in the system model. The forecasted variables are inside the model of the system and outcome in the current value in the system model, demonstrating consistency with the projections. The provided data, from the period, presents a new set of data points which is further used for the updating of the least square coefficients in the forecasting equations. The next period use the new coefficients into the model. The disadvantage of least square is the sensitivity it offers to outliers. A couple of unusual point's presence may cause tremendous skew in the final result.

3.2 Recursive Least Square (RLS)

For system identification of adaptive control and filtering, recursive least square method is an attractive alternative for the reduction of the computational load which is part and parcel of least square method. Recursive least square is a confined form of Kalman filter.

For accurate description of behavior of systems least square method was introduced. In least square estimation, a linear model of unknown parameter is selected that the sum of the square of the errors between observed values and computed value is lowest. If parametric values of system changes rapidly, cyclic resetting of the estimation method can potentially confine the latest values of the parameter. A special heuristic but efficient approach is used for the parameters which varies continuously but at a snail's pace.

The concept of forgetting is that in which preceding data is step by step discarded in favor of further current information. In least square methodology, forgetting can be examined as gaining less consideration to previous data and more to recent data. The classical recursive least square was not capable of tracking parametric changes as its covariance vanishes to zero with respect to time. Recursive least square with forgetting has been widely used in tracking and estimation of parameters which are varied with time in several fields of engineering. With the poor excitation of system, this scheme can show the way to the covariance wind up problem. Many techniques have been suggested to tackle covariance windup issues. Several researchers suggested binding the expansion of covariance matrix by introducing an upper bound. Time varying forgetting factor is a renowned scheme used by Fortes cue.

RLS for the q-th order can be expressed as,

Q = Order of the filter.

λ=Forgetting factor.
δ=initilization of Q(0).

Initialization,

w(n)=0,
x(k)=0,where k= -q ,………...,-1
d(k)=0,where k= -q ,………...,-1
Q(0)= δ^(-1) I,where I is an identity matrix.

For n = 1, 2…

$$x(n) = \begin{bmatrix} x(n) \\ x(n-1) \\ \vdots \\ x(n-q) \end{bmatrix} \quad (13)$$

$$a(n) = d(n) - x^T(n)w(n-1) \quad (14)$$

$$g(n) = \frac{Q(n-1)x(n)}{\lambda + x^T(n)Q(n-1)x(n)} \quad (15)$$

$$Q(n) = \lambda^{-1}Q(n-1) - g(n)x^T(n)Q(n-1) \quad (16)$$

$$w(n) = w(n-1) + a(n)g(n) \quad (17)$$

4. RESULTS & DISCUSSION

The parameters of recursive least square and least mean square methods are dependent on time. For the allowance of adjustment of the parameters, a smart solution is the addition of forgetting factor in the revising of the equation. Forgetting factor gives a lesser amount of significance while estimation takes place in order that parametric values have more dependency on recently occurred events. The case in which it might be wished to give less weight to the previous data is either because it is believed that structural change may have occurred in the model or somehow relevance of recent observations are more relevant to the observations made previously. Addition of forgetting factor has a tendency of making the coefficients equation differs with time and result is more dependent on recent outcomes.

In above paragraph we discussed about the comparison of LST and LMS, which of them provide us with better identification with minimum errors. In fig.2 we can see the error response of alpha (LMS) approximately equal to 9.66%, in fig.3 we can see that beta (LMS) is approximately equal to 4.45% and Gamma (LMS) in fig.4 is approximately equal to 8.19%. On the other hand we can observe the error response for alpha (RLS) in fig.5 is approximately equal to 3.61%, for beta(RLS) in fig.6 approximately equal to 2.823% and for Gamma (RLS) in fig.7 approximately equal to 4.47%. According to the data acquired by both algorithms, it is obvious that RLS provides us with better identification as it produces minimum error if compared with LMS algorithm.

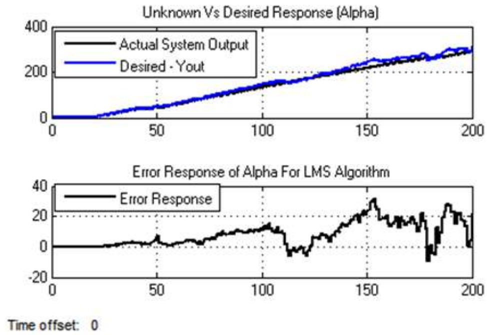

Figure 2: Approximate and error response for LMS (Alpha)

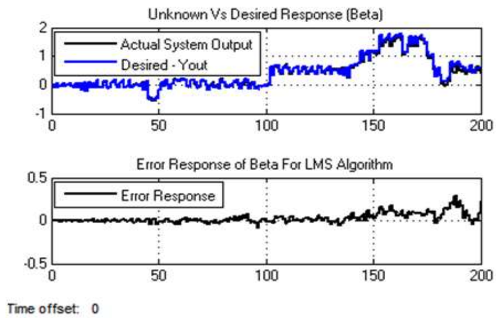

Figure 3: Approximate and error response for LMS (Beta)

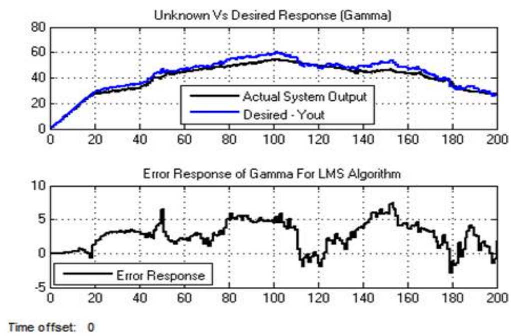

Figure 4: Approximate and error response for LMS (Gamma)

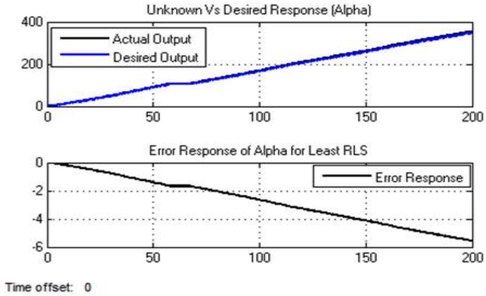

Figure 5: Approximate and error response for RLS (Alpha)

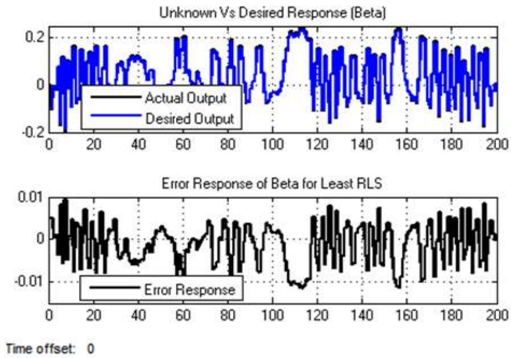

Figure 6: Approximate and error response for RLS (Beta)

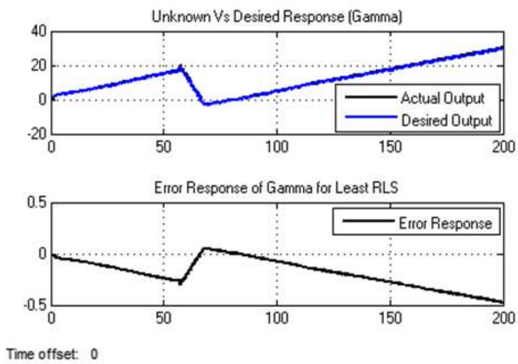

Figure 7: Approximate and error response for RLS (Gamma)

CONCLUSION

After analyzing the results of both the algorithm, we can conclude that Recursive Least square (RLS) provides us with better estimation of an unknown model as compared to Least mean square (LMS) algorithm. We have analyzed the results of a fighter plane mathematical model with respect to RLS and LMS to identify which one of them provides us with minimum error. So in the end, error generated by RLS in determining the exact model was less than LMS clearly showing that it provides us with better estimation than LMS for given task.

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